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Transition to large aspect ratio convection

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We describe experiments with aspect ratio Γ in the range $44 \le \Gamma \le 90$ that characterize the transition from moderate- to large-Γ convection. Up to Γ=60, the flows are steady until well above the convective onset. For $\Gamma \ge 60$, we find a slow periodic state near the convective onset. A clear rise in the noise floor is seen just above onset for Γ=70. For $\Gamma \simeq 90$, the flows are irregular and time dependent arbitrarily close to the convective onset, and the amplitude of the noise signal is proportional to the Nusselt number.

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At the onset of Rayleigh-Bénard convection, an experimental cell becomes filled with rolls with a characteristic wave number near $\alpha \simeq 2\pi/d$, where d is the fluid layer height. As the cell's lateral size L increases, more rolls are needed to fill the cell, and the number of degrees of freedom grows. L is characterized by the aspect ratio $\Gamma = L/d$. By gradually increasing Γ, we may study how a prototypical pattern-forming system changes at a bifurcation as more and more modes become active. A fundamental question is, what happens near onset as the aspect ratio becomes large?

Theory for a horizontally infinite system predicts [1,2] a domain in parameter space where straight parallel rolls are stable. The relevant parameters are the Rayleigh number R (the bifurcation parameter [3]), the Prandtl number Pr, and α. The stable domain is called the Busse balloon. Visualizations with up to 36 rolls [4] show that the pattern at the onset of convection for a low Pr fluid consists of stationary rolls, which may also be spatially ordered. Previous measurements [5] in a very large cylindrical container [Γ =(radius)/d = 57] indicated unexpected noisy time dependence which appears to be associated with the large aspect ratio. More recently, experiments in even larger (Γ=86,78) cylindrical containers [6,7] have shown both large rotating spirals and spiral chaos.

Here, we report experiments using a variable height cell which focus on the crossover between moderate and large Γ. The point of these experiments is (i) to characterize the states near onset as Γ is varied from moderate to large values, and (ii) to clarify whether the noisy states at large Γ show transient time dependence or stationary chaos.

Convection begins when R exceeds a critical value R_c and we define $r \equiv R/R_c$ [3]. A measure of the convective strength is the Nusselt number N defined as the total heat flux across the layer normalized by the preconvective heat flux: $N = 1$ below the onset of overturning convection, and N rises continuously above 1 as r increases above 1. The Prandtl number in the present experiments is Pr=1.30, which is comparable to that of gases or normal liquid helium.

The convecting fluid for these experiments is a ^{3}He-superfluid-^{4}He mixture [3]. The container geometry is rectangular with horizontal dimensions of $L_x = 2.284$ cm and $L_y = 1.013$ cm. The apparatus [8] allows for the *in situ* adjustment of d from 1.006 cm to 0. By Γ we mean L_x/d. The experiments are made by fixing the temperature at the bottom boundary of the fluid layer at $T = 2.086$ K and applying a steady heat current to the top plate fluid boundary [9]. We measure ΔT with bridge circuits and germanium resistance thermometry. We refer to the time evolution of ΔT by $\delta T(t)$. The sidewalls of the experiment are stainless steel, which has a thermal conductivity two to three times smaller than that of the mixture.

The tight coupling between the temperature and concentration causes superfluid mixtures to convect as a standard classical single-component fluid. Recently [10], we calculated and measured the size of superfluid effects which modify the standard dynamics of a Boussinesq fluid and showed that the only superfluid effects near onset are a small increase in R_c and a decrease in α. The number of convection rolls in an experiment $n \simeq \Gamma\alpha/\pi$ is the most direct measure of the size of the system. In the following, we give both Γ and the estimated number of rolls.

Superfluid mixtures and the cryogenic environment offer several advantages. First, the metal bounding plates have heat capacities, relative to the fluid, which are orders of magnitude smaller than what can be achieved at room temperatures. Consequently, cryogenic experiments are unmatched at identifying time dependence in thermal signals. Specifically, the ratio of heat capacities of equal volumes of fluid and copper is 1.4×10^3 if the fluid is liquid helium, 1.2 if the fluid is room-temperature water, and 7.3×10^{-3} if the fluid is an ideal gas at room temperature and a pressure of 30 bar. Second, cryogenic experiments allow very high precision temperature measurements. Third, even for large Γ, ΔT_c is only a few mK [10], guaranteeing that the system remains Boussinesq [11]. Specifically, as the mean temperature is increased towards the superfluid transition temperature, at fixed d,

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ΔT_c decreases rapidly. Finally, the thermal conductivity of the horizontal boundaries is many orders of magnitude larger than that of the fluid. Consequently, each of these boundaries may be considered isothermal.

Figure 1 shows the important features of time series for δT in large-aspect-ratio containers ($\Gamma = 70$ in this case) near onset. Part (a) characterizes the instrumental noise level just below convective onset. Part (b), characteristic of convection in the large-Γ regime, has a slightly larger noise amplitude than the preconvective state, part (a). The slow coherent oscillations in part (c) occur during the crossover from moderate to large Γ. All time series are digitized at a 2.5-Hz acquisition rate and Fourier transformed to obtain a power spectrum.

To quantify the fluctuation amplitude, δT about ΔT, we calculate the total power, defined as

$$P = \int_{f_c}^{f_{\mathrm{Ny}}} \mathcal{P}_{\mathrm{PSD}} df, \quad (1)$$

where $\mathcal{P}_{\mathrm{PSD}}$ is the power spectral density, f_{Ny} is the Nyquist frequency, and f_c is a lower frequency cutoff used to eliminate the effect of small drifts. Typical power spectra, to be shown elsewhere, are flat at low frequencies, and fall off rapidly at higher frequencies.

Figures 2–4 show plots of N and P versus r. P is normalized by P_b, the value of P below onset. The dots denote N and are labeled by the left axis; the squares denote P/P_b and are labeled by the right axis. Filled squares are points with only broadband fluctuations; cf. Figs. 1(a) and 1(b), and open squares show points with periodic oscillations, cf. Fig. 1(c).

Figure 2 (Γ=44, $n = 36$) shows the behavior expected from steady convection. The Nusselt number is 1 in the absence of overturning convection and rises sharply above 1 when convection begins. P/P_b shows no detectable change over $0.6 \leq r \leq 2$. Near $r = 2$, the system undergoes a long complex transient, which leads to a new steady state at lower N. This transition is symptomatic of an encounter with the skewed varicose boundary of the Busse balloon [1,4,12]. We conclude that at $\Gamma = 44$ we see the regular behavior at and above onset as expected from theory [1,2].

We contrast the regular behavior for $\Gamma = 44$ with re-

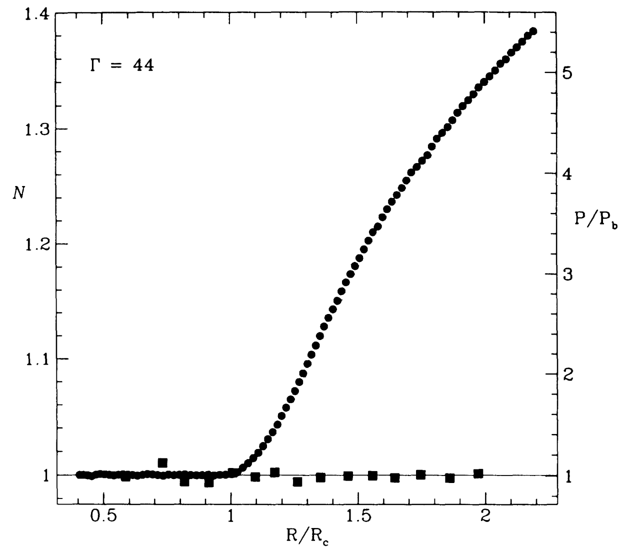

FIG. 2. Nusselt number N and noise power P/P_b vs r for $\Gamma = 44$. The dots give N and go with the scale on the left. The squares give P/P_b and go with the scale on the right.

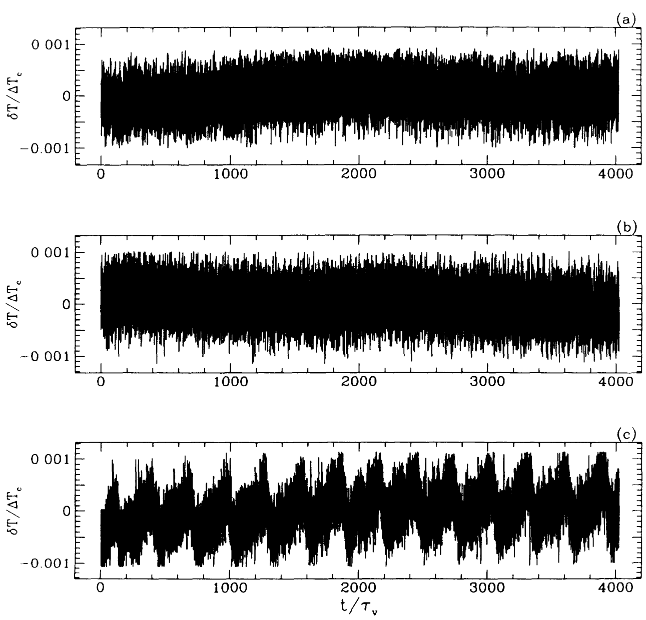

FIG. 1. Time series at $\Gamma = 70$ showing (a) the instrumental noise, (b) the rise in noise level for convective states with large enough Γ, and (c) the slow periodic state seen for aspect ratios between moderate and large. For $r \equiv R/R_c = 0.917$, 1.429, and 1.368, respectively.

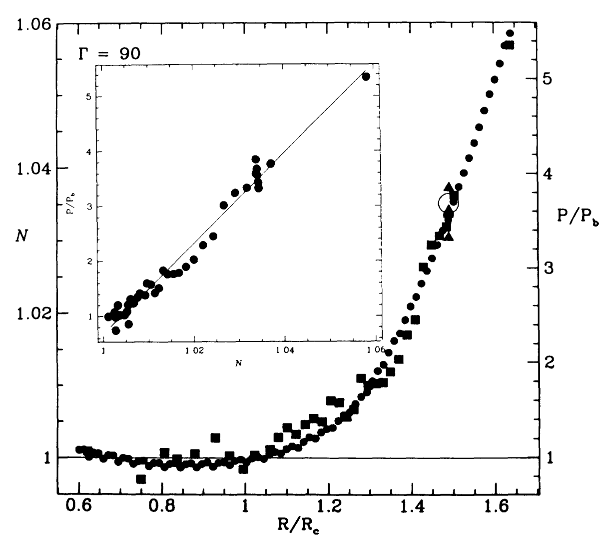

FIG. 3. Nusselt number and noise power vs r for $\Gamma = 90$. The dots and squares are as in Fig. 2. The five triangles at $r = 1.49$ are the power calculated from consecutive $1\tau_{\mathrm{hl}}$ segments of a run unperturbed for $5\tau_{\mathrm{hl}}$. The center of the large open circle gives the power calculated from the entire unperturbed run. Inset: N vs P/P_b; the line is a least-squares fit to the data for $r > 1$.

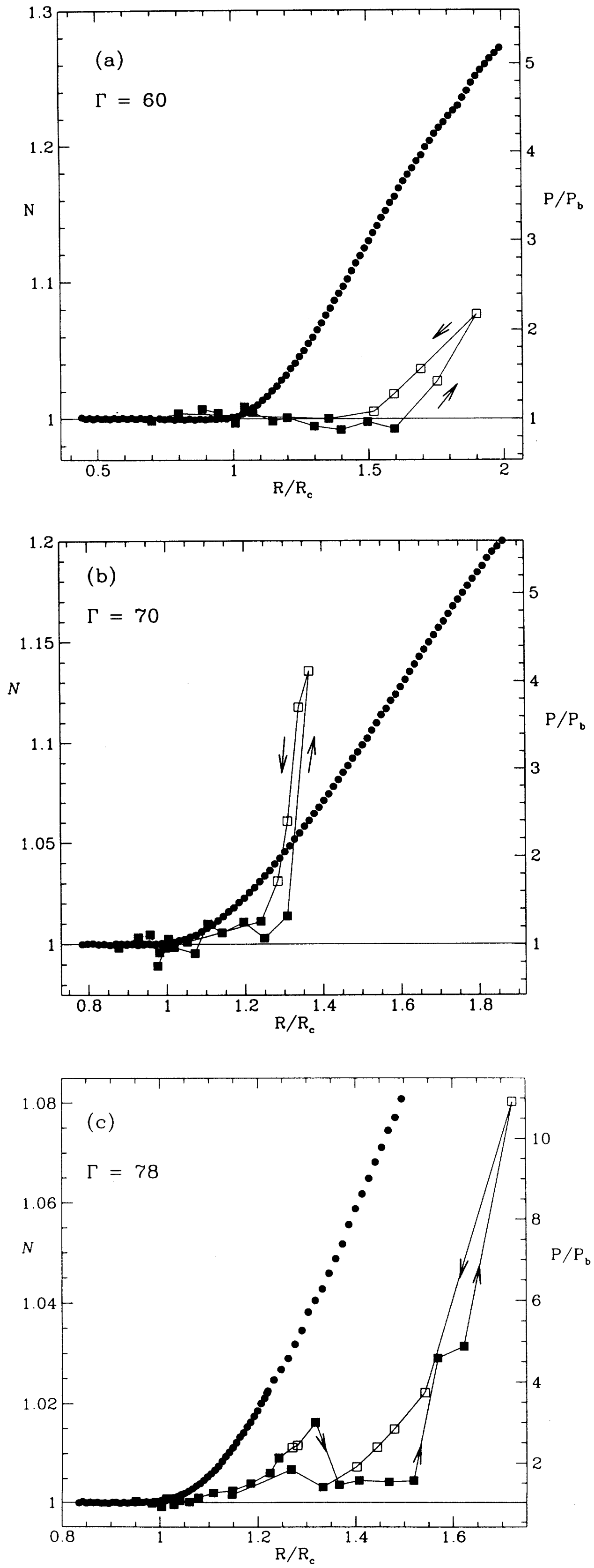

FIG. 4. Nusselt number and noise power vs r: (a) $\Gamma = 60$; (b) $\Gamma = 70$; (c) $\Gamma = 78$. The dots and solid squares are as in Fig. 2. Open squares denote the presence of the periodic state. Arrows indicate data taken when raising or lowering r.

sults for $\Gamma = 90$ ($n = 60$), where irregular time dependence exists as soon as overturning convection begins. P/P_b and N for $\Gamma = 90$ are shown versus r in Fig. 3. Unlike the regular case, P/P_b rises above unity in tandem with the Nusselt number. The correlation between N and P/P_b is essentially linear, as shown in the inset. The rise in noisy spectral power is qualitatively similar to earlier observations [5]. Presumably, this state is associated with the formation of defects and the corresponding mean flows [7,13,14].

In Fig. 4, data for $\Gamma = 60$, 70, and 78 show the key features of the transition regime. For $1 \leq r \leq 1.7$, $\Gamma = 60$ (45 rolls) resembles lower Γ's and P/P_b remains at unity. At $r = 1.7$, however, a new periodic state appears. Figure 1(c) shows an example of this periodic state which we find for $60 \leq \Gamma \leq 83$. Note that the period is about $300\tau_v$. This is an order of magnitude smaller than the period reported by Bodenschatz *et al.* [6] for large spirals in non-Boussinesq convection. For comparison, the horizontal diffusion times are $\tau_{\rm hl} = 4900\tau_v$ and $\tau_{\rm hs} = 960\tau_v$ for the long and short horizontal directions, respectively.

Although the periodic state appears in the crossover regime, it does not appear to be related to the noisy state, since the noisy state appears very near onset by the time Γ reaches 70 ($n = 50$). In Fig. 4(b), P/P_b rises slowly above unity, due to the noisy state (solid squares). In this case, the periodic state occurs at $r \simeq 1.3$.

We now turn to $\Gamma = 78$ ($n = 53$). Except for a narrow periodic window (open squares near $r = 1.25$), the signal remains noisy, i.e., no periodic features are present. In the vicinity of $r = 1.5$, the noise power increases rapidly, and a periodic state eventually emerges slightly above $r = 1.7$ (open squares). On decreasing r, the periodic state persists down to $r = 1.4$, where the noisy state is seen once again. We did not detect the periodic state at all for $\Gamma = 90$, but we could not exceed $r = 1.65$ because of cooling power limitations. Note that Morris *et al.* [7] report spiral convection above $r = 1.4$.

An important question is whether the noisy states seen at large Γ are transient or persistent. Clearly, the horizontal diffusion time in the long direction $\tau_{\rm hl} = \Gamma^2/\tau_v$ is the important time scale here. Each square point in Figs. 2–4 represents a time-series approximately $1\tau_{\rm hl}$ long. This is not long enough to completely guarantee that measurements made after raising the heat current are relevant to a fully relaxed system. Unfortunately, the transfer of helium into the Dewar every two days produces a perturbation comparable to small changes in the heat current, so we cannot let the experiment sit undisturbed indefinitely. However, to partially address this issue, we have taken data undisturbed for the full two days between transfers, and then calculated P from consecutive $1\tau_{\rm hl}$ segments of the data. The triangles in Fig. 3 are data for P from five such segments out of a time series of total length $5\tau_{\rm hl}$. The center of the large open circle shows the average from the entire run. Segments 1, 3, and 4 fall nearly on top of each other; segment 2 is above the average; and segment 5 is below the average. It is also relevant that we observe fluctuations with $P/P_b > 1$ for weeks at a time (15–$30\tau_{\rm hl}$); after a transfer perturbation, the system will relax back into the same oscillatory or noisy state with the same mean N and P/P_b. If the noisy state were transient with a decay time proportional to τ_{hl}, then it would be difficult to understand why the change from $\Gamma = 60$ to $\Gamma = 70$ shows a noticeable effect on the noisy state. The evidence is consistent with the system being in a statistically stationary state.

We have identified several key features of the states just above the onset of convection when Γ is large. We find three regimes: (i) At smaller Γ, only regular behavior occurs. (ii) At moderate Γ, the fluctuation amplitude grows above onset, presumably due to defects and the resulting mean flows [7,13,14], and there is a hysteretic transition to coherent oscillations at $r = 1.3$–1.6, the exact point depending on Γ. The periodic state does not have a clear corollary to any previously reported state, although rotating spirals of much longer period have been reported by Bodenschatz *et al.* [6] for non-Boussinesq convection in a cylindrical container. In this regard, the rectangular geometry of the present experiments may play a role. (iii) At larger Γ, the fluctuation amplitude increases continuously with r above onset and is correlated with the Nusselt number. This state is presumably the same as that reported by Ahlers and Behringer [5]. A noise level greater than instrumental first occurs for $\Gamma \simeq 70$. This observation is worth emphasizing because the noisy state is absent at $\Gamma = 60$, but clearly present at $\Gamma = 70$, indicating a relatively sharp onset of the noisy state. The present experiments do not show any tendency for the noisy state to decay.

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[3] The Rayleigh number $R = |\alpha_{p,\mu_4}|gd^3\Delta T/\nu\chi_{\mathrm{eff}}$, and the Prandtl number $\mathrm{Pr}=\nu_n/\chi_{\mathrm{eff}}$. ΔT is the temperature difference across a fluid layer of height d. The fluid parameters α_{p,μ_4}, χ_{eff}, and ν are, respectively, the thermal expansion coefficient at constant pressure and ^{4}He chemical potential μ_4, an effective thermal diffusivity, and the kinematic viscosity; g is the acceleration of gravity. Also, $\nu_n = (\rho/\rho_n)\nu$, where ρ and ρ_n are, respectively, the total and normal fluid densities. The vertical thermal diffusion time $\tau_v = d^2/\chi_{\mathrm{eff}}$. The ^{3}He molar concentration is $X = 0.0122$. The mean temperature is $T = 2.086$ K, making $\mathrm{Pr}=1.30$ and $\chi_{\mathrm{eff}}=1.27\times10^{-4}$ cm^2/sec.

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